Key

Math 4

Name _____

Date _____

U4 L1 I1 Part 1 - Trigonometric Simplification

In this investigation, you will be working towards the following learning goals:
I can use and define the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent
I can use the fundamental trigonometric identities to simplify expressions and verify equivalences

Below are the **six** trigonometric functions.

Cosecant w/ sine & secant w/ cosine!

1. sine (sin)
2. cosine (cos)
3. tangent (tan)
4. cosecant (csc)
5. secant (sec)
6. cotangent (cot)

The six trigonometric functions are related in the following way:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

A. Write $\sin x$, $\cos x$, and $\tan x$ in terms of secant, cosecant, and cotangent.

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

You will need to have the above six definitions memorized in order to succeed in this lesson.

B. The functions of secant, cosecant, and cotangent are called *reciprocal functions*. Explain why.

They equal the reciprocal of cosine, sine + tangent, respectively.

C. Previously, we have learned that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. This relationship is an example of an ***identity***. An identity is *an equation that is true for all values of the variable (as defined by the domain)*

Based on the definition of $\cot\theta$ and the identity for $\tan\theta$, write an identity for $\cot\theta$

$\star \sin^2 x = (\sin x)^2$ $\star$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = 1 \cdot \frac{\cos\theta}{\sin\theta} = \boxed{\frac{\cos\theta}{\sin\theta}}$$

Some Useful Trig Identities:

1. $\sin^2 x + \cos^2 x = 1$ (Pythagorean Identity) $\rightarrow \sin^2 x = 1 - \cos^2 x$, $\cos^2 x = 1 - \sin^2 x$

2. $1 + \tan^2 x = \sec^2 x$ $\rightarrow \tan^2 x = \sec^2 x - 1$, $1 = \sec^2 x - \tan^2 x$

3. $1 + \cot^2 x = \csc^2 x$ $\rightarrow \cot^2 x = \csc^2 x - 1$, $1 = \csc^2 x - \cot^2 x$

Examples: Simplify.

C1. $\sec x \bullet \sin x \bullet \cot x =$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} = \boxed{1}$$

C2. $\sec x - \sec x \bullet \sin^2 x =$

$$= \sec x (1 - \sin^2 x)$$

$$= \sec x (\cos^2 x)$$

$$= \frac{1}{\cos x} \cdot \cos^2 x = \boxed{\cos x}$$

C3. $\dfrac{\cos x}{\cot x} =$

$$\frac{\cos x}{\frac{\cos x}{\sin x}} = \cos x \cdot \frac{\sin x}{\cos x} = \boxed{\sin x}$$