

U4 L1 I1 Part 1 - Trigonometric Simplification

In this investigation, you will be working towards the following learning goals:

- I can use and define the six trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent
- I can use the fundamental trigonometric identities to simplify expressions and verify equivalences

Below are the **six** trigonometric functions.

- | | |
|------------------|--------------------|
| 1. sine (sin) | 4. cosecant (csc) |
| 2. cosine (cos) | 5. secant (sec) |
| 3. tangent (tan) | 6. cotangent (cot) |

Cosecant w/ sine
+
Secant w/ cosine!

The six trigonometric functions are related in the following way:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

A. Write $\sin x$, $\cos x$, and $\tan x$ in terms of secant, cosecant, and cotangent.

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

You will need to have the above six definitions memorized in order to succeed in this lesson.

B. The functions of secant, cosecant, and cotangent are called *reciprocal functions*. Explain why.

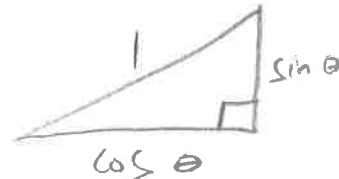
They equal the reciprocal of cosine, sine + tangent, respectively.

C. Previously, we have learned that $\tan \theta = \frac{\sin \theta}{\cos \theta}$. This relationship is an example of an **identity**. An identity is an equation that is true for all values of the variable (as defined by the domain)

Based on the definition of $\cot \theta$ and the identity for $\tan \theta$, write an identity for $\cot \theta$

$\sin^2 x = (\sin x)^2$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{\sin \theta}{\cos \theta}} = 1 \cdot \frac{\cos \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta}$$



Some Useful Trig Identities:

$$1. \sin^2 x + \cos^2 x = 1 \quad \begin{matrix} \rightarrow \sin^2 x = 1 - \cos^2 x \\ \rightarrow \cos^2 x = 1 - \sin^2 x \end{matrix} \quad \text{(Pythagorean Identity)}$$

$$2. 1 + \tan^2 x = \sec^2 x \quad \begin{matrix} \rightarrow \tan^2 x = \sec^2 x - 1 \\ \rightarrow 1 = \sec^2 x - \tan^2 x \end{matrix}$$

$$3. 1 + \cot^2 x = \csc^2 x \quad \begin{matrix} \rightarrow \cot^2 x = \csc^2 x - 1 \\ \rightarrow 1 = \csc^2 x - \cot^2 x \end{matrix}$$

Examples: Simplify.

c1. $\sec x \cdot \sin x \cdot \cot x =$

$$\frac{1}{\cancel{\cos x}} \cdot \frac{\cancel{\sin x}}{1} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} = \boxed{1}$$

c2. $\sec x - \sec x \cdot \sin^2 x =$

$$= \sec x (1 - \sin^2 x)$$

$$= \sec x (\cos^2 x)$$

$$= \frac{1}{\cos x} \cdot \cos^2 x = \boxed{\cos x}$$

c3. $\frac{\cos x}{\cot x} =$

$$\frac{\cos x}{\frac{\cos x}{\sin x}} = \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} = \boxed{\sin x}$$